FURROW INFLOW AND INFILTRATION VARIABILITY IMPACTS ON IRRIGATION MANAGEMENT

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ABSTRACT

Furrow-to-furrow infiltration variability causes non-uniform water absorption rates, furrow stream advance rates, and runoff rates from the furrow tail end. Unevenly set inflow rates to furrows compound these latter two non-uniformities. In order for an irrigator to ensure adequate water advance on a desired portion of furrows, the average inflow rate must be increased. To ensure adequate water application to a desired portion of the furrows, the application time must be extended. Thus, inflow and infiltration variability result in excess water application and reduced irrigation water use efficiency. Models, based on Gaussian distributions of inflow and infiltration, are presented which relate excess furrow irrigation applications to these variabilities. **KEYWORDS.** Irrigation, Furrows, Surface irrigation, Infiltration, Variability.

INTRODUCTION

With surface irrigation, the soil surface serves both as a medium into which water is absorbed and as a conduit to convey water across the field. Consequently, infiltration variability will affect both the relative infiltration rate at a location, the surface water distribution rate and, thus, the infiltration opportunity time, IOT, for other locations further from the inlet. Both the infiltration rate and the IOT determine the net water application depth at a location.

The large spatial variability in infiltration has been established by many studies (Vieira et al., 1981; Sisson and Wierenga, 1981; Izadi and Wallender, 1985). Infiltration coefficients of variation (CV = standard deviation/mean) commonly vary between 20% and 60%. Bautista and Wallender (1985) calculated CVs of 180-minute infiltrated volume and final infiltration rate of 53% and 21%, respectively, on 30 one-meter long subsections of a furrow. Tarbonton and Wallender (1989) calculated a furrow cumulative infiltration CV of 24% with measurements from a rectangular grid of 100 one-meter-long blocked furrow sections. Trout and Mackey (1988) measured furrow-to-furrow final infiltration rate variability of complete furrows on 50 fields in three states. Coefficients of variation ranged from 10% to 100% and averaged 25%.

Farmers set furrow inflow rates so that water will advance across the field in a desired amount of time without producing excessive tailwater runoff. Although the usual intent is to set inflow rates uniformly on a set of equal-length furrows, Trout and Mackey (1988) measured inflow rate variability (CV) of 15% for siphon tube, 25% for gated pipe, and 29% for feed ditch water application techniques. This furrow-to-furrow inflow variability combines with the furrow-to-furrow infiltration rate variability to produce even greater variability in furrow stream advance rates and tailwater runoff.

Furrow irrigators are aware of this stream advance variability. Their response is to increase inflow rates to ensure that adequate advance rates are achieved on a large portion of the furrows. This results in an increase in runoff rates.

Although farmers may not be directly aware of the effect of infiltration variability on water application, they are aware that the crops in certain locations on their fields show signs of water stress earlier than at other locations. Their response is to over-irrigate by extending the application time to limit the stressed area to an acceptable portion of the field. Extending the application time increases both runoff and deep percolation losses.

The objective of this article is to analyze the impact of furrow-to-furrow inflow and infiltration variability on irrigation management and efficiency. Statistical models are used to quantify the impacts.

In this work, only the effects of variability in infiltration and inflow rates among furrows is considered. A uniform infiltration relationship down individual furrows is assumed. Although infiltration variability among sub-sections of individual furrows will generally be larger than variability among whole furrows (Tarbonton and Wallender, 1989), furrow-to-furrow variability creates the non-uniformity in stream advance and tailwater runoff on which irrigators base many of their operational decisions. Rayej and Wallender (1987) describe a hydraulic model which can determine the effects of infiltration variability along a furrow on advance rates and the distribution of water application on individual furrows.

The term infiltration (both rate and cumulative) refers to the soil's capacity to absorb water. Water application or absorption refers to the amount of water absorbed at a location, which is dependent on both the soil infiltration capacity and the IOT at the location. Gross water application refers to the total inflow to a furrow or field.

The analyses in this article assume that infiltration variability remains constant with opportunity time, or that the relative amount of variability is the same early and late in the irrigation. Thus, the infiltration CV will not vary with IOT. Although the variability will often change with infiltration time (Bautista and Wallender, 1985), time-dependent variability complicates the analyses but will generally not change the primary conclusions.

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INFILTRATION DISTRIBUTION MODEL

Distributed data such as spatially varying infiltration or irrigation water application, can be described with a cumulative distribution (or frequency) curve such as that shown in figure 1. The vertical axis represents soil infiltration capacity (or water application depth) and the horizontal axis indicates the portion of the total sample (field) with lower capacity (less depth). Equivalently, the horizontal axis represents the probability of lower infiltration for each location (furrow). Such a curve can be generated by subdividing a field into many sub-areas and arranging and plotting the samples in increasing order by infiltration rate or water application. The orientation shown depicts depth of water infiltrated downward from a soil surface over a certain portion of the field area. For example, 90% of the total area (furrows) in figure 1 had a cumulative infiltration depth less than I_p. Cumulative distributions are discussed in basic statistics texts. Hart and Reynolds (1965), Warrick (1983), Till and Bos (1985), and Seginer (1987) discuss their application to irrigation system performance. Note that most irrigation applications reverse the horizontal axis in figure 1 to depict the probability of exceeding a desired amount (one minus the actual cumulative probability).

Researchers have used several mathematical models to describe irrigation application distributions (Warrick, 1983). Infiltration into small areas is often found to be normally or log-normally distributed (Warrick and Nielsen, 1980; Sharma et al., 1983; Vieira et al., 1981; and Jaynes and Clemmens, 1986). Trout and Mackey (1988) found that the distribution of final infiltration rates for whole furrows could be described by a normal distribution. A normal (Gaussian) distribution was used in these analyses.

The cumulative distribution or probability, P(z), for normally distributed data is described mathematically by

$$P(z) = (2\pi)^{-0.5} \int_{-\infty}^{z} \exp(-v/2)\, dv \qquad (1)$$

where

z = the standardized variate = $(x-\bar{x})/s$
= $(x-\bar{x})/(CV\cdot\bar{x})$,
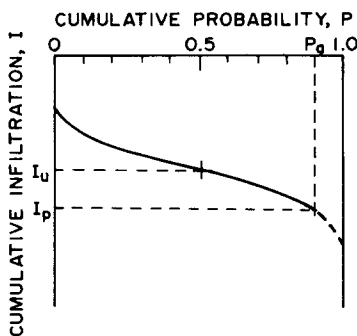$\bar{x}$ = mean,
s = standard deviation, and

Figure 1–Typical cumulative distribution of furrow infiltration ($CV_I = 0.25$).

CV = coefficient of variation = $s/\bar{x}$.

Equation 1 cannot be solved explicitly, so the solution is given in tabular form in statistics texts. For irrigation management purposes, the fraction of the field which is to be left deficient of a requirement, P, is chosen and the value of z such that P(z) = P is determined from the normal distribution table. Once z is determined, values of the distribution can be calculated from the definition of the standardized variate. For example, to determine the infiltration depth which is exceeded on 1-P portion of the furrows, I_p

$$z_p = \frac{x-\bar{x}}{CV\cdot x} = \frac{I_p - I_u}{CV_I\cdot I_u} \qquad (2)$$

where z_p is the standardized variate for the desired probability, P; CV_I is the coefficient of variation of cumulative infiltration; and I_u is the mean infiltration depth.

Solving for I_p,

$$I_p = z_p \cdot CV_I \cdot I_u + I_u \qquad (3)$$

Expressed relative to the mean,

$$I_p / I_u = 1 + CV_I \cdot z_p \qquad (4)$$

EFFECT OF INFILTRATION VARIABILITY ON INFLOW RATE

A typical cumulative distribution of infiltration among furrows is shown in figure 1. If a uniform inflow rate is applied to all furrows, water will advance most rapidly on those furrows with the lowest infiltration rate (left side of the figure) and slowest on those with the highest infiltration rate. Irrigators usually try to set inflows at a rate sufficient to complete advance by some desired time, t_a, at least on a large portion of the furrows, P (or equivalently, with probability P on each furrow). Thus, the inflow rate, Q, (assumed equal on all furrows) must be adequate to achieve advance on a furrow with cumulative infiltration I_p at time t_a. If infiltration were uniform, the cumulative infiltration of all furrows would be equal to the average, I_u, so the inflow rate would need to be sufficient to complete advance on a furrow with cumulative infiltration I_u at time t_a. The difference between these two inflow rates represents the excess inflow rate required to meet the advance criteria (P and t_a) due to infiltration variability among furrows.

The inflow volume required to complete advance is the volume absorbed during advance, d, plus the volume stored on the surface in the furrow, S. Since inflow volume is inflow rate times advance time and assuming surface storage on all furrows at advance completion is equal, the ratio of inflows required to complete advance by t_a on two furrows which absorb volumes d_u and d_p during advance is given by

$$\frac{Q_p \cdot t_a}{Q_u \cdot t_a} = \frac{d_p + S}{d_u + S} = \left(\frac{d_u}{d_u + S}\right)\left(\frac{d_p}{d_u} - 1\right) + 1 \qquad (5)$$

where Q_p is the inflow rate required to complete advance by t_a on a furrow which absorbs d_p volume of water during advance; and Q_u is the inflow rate required on a furrow with d_u water absorption.

Since the criteria is to set inflow rates to complete advance by t_a, then the average IOT along the furrow during advance would be equal on any furrow which meets the criteria (i.e., the targeted furrow with inflow Q_p and the average furrow with inflow Q_u). Since the average IOTs are equal, the volume absorbed by each furrow is proportional to the cumulative infiltration and the ratio of the volumes applied, d_p/d_u, would equal the ratio of the cumulative infiltration of the furrows, I_p/I_u. Substituting this equality and equation 4 into equation 5 gives

$$\frac{Q_p}{Q_u} = \left(\frac{d_u}{d_u + S}\right)\left(\frac{I_p}{I_u} - 1\right) + 1 = \left(\frac{d_u}{d_u + S}\right) CV_I \cdot z_p + 1 \quad (6)$$

This inflow ratio represents the excess inflow rate and, thus, gross application required due to the effect of infiltration variability among furrows on advance times. The excess inflow rate results in P portion of the furrows completing advance by the target advance time, t_a. It also results in water loss directly attributable to infiltration variability. The loss is primarily to runoff from furrows

EFFECT OF COMBINED INFLOW AND INFILTRATION VARIABILITIES ON INFLOW RATE

In the preceding analysis, the flows were assumed to be uniformly applied to all furrows. However, furrow inflows are not uniform. Trout and Mackey 1988) found that most furrow-to-furrow inflow variability is not related to the furrow infiltration, which would indicate farmer adjustment in response to observed advance or runoff differences, but rather to a random inability of the irrigator to set water uniformly. The result of the combination of inflow and infiltration variabilities in furrow irrigation is even greater variability in the water advance. Thus, at a given average inflow rate, a larger portion of the furrows will not complete advance at the required advance time so the average inflow rate must be further increased.

The effect of the combination of the two variabilities on advance can be calculated. As in the previous analysis, advance will be complete on a furrow if the inflow volume to a furrow at the desired advance time exceeds the amount absorbed, d, plus surface storage, S. The probability that inflow is larger than d+S, or equivalently, that $(Q \cdot t_a)/(d+S) > 1$ on a certain portion of the furrows can be calculated from the confidence interval, CI, of a ratio (Cochran, 1963)

$$CI = \left(\frac{\overline{Q} \cdot t_a}{d_u + S}\right) \frac{(1 - t^2 r CV_D CV_Q/N) \pm t\sqrt{CV_Q{}^2/N + CV_D{}^2/N - 2r CV_Q CV_D/N - t^2 CV_Q{}^2 CV_D{}^2(1 - r^2)/N^2}}{(1 - t^2 CV_D{}^2/N)} \quad (7)$$

with infiltration less than I_p and, thus, faster advance than the target advance. This runoff loss is in addition to that which results from constant inflows and decreasing infiltration rates after advance is complete.

Surface storage decreases the required excess inflow rate below that predicted if only infiltration variability were considered. Equations 5 and 6 assume surface storage is equal with both inflow rates, Q_p and Q_u. In fact, surface storage increases somewhat with flow rate. This refinement would tend to reduce the influence of surface storage and cause equation 6 to slightly underestimate the required excess inflow rate.

EXAMPLE
Completed advance is desired by time t_a on 90% of the furrows (P = 0.9, z_p = 1.28) on a field with an infiltration coefficient of variation among furrows of 0.25 (depicted in fig. 1). If the furrow surface storage volume at advance completion is estimated to be $4m^3$ and the average infiltrated volume at t_a is $20m^3$, then by equation 6, Q_p/Q_u = (20/(20+4))·0.25·1.28+1 = 1.27 or the average inflow rate must be increased by 27% to meet the advance criteria just due to infiltration variability. If surface storage were not considered, the predicted excess application would have been 1.32 (= I_p/I_u).

where

t	= the Student's statistic (1-tailed) based on the number of furrows, N, and the desired probability level, P,
r	= the correlation coefficient between inflow and infiltration (plus storage),
$\overline{Q}$	= the average furrow inflow rate,
CV_Q	= the inflow coefficient of variation, and
CV_D	= the CV of the sum of the amount absorbed and surface storage (d+S) at the desired advance time.

For this analysis, we wish to determine the required ratio of the average inflow to the average amount absorbed plus storage, $(\overline{Q} \cdot t_a)/(d_u + S)$, such that the lower confidence bound for each furrow (i.e., N=1) for the chosen probability is one (i.e., $Q \cdot t_a \geq d + S$ with the chosen probability on every furrow or equivalently, the portion of furrows which have completed advance by time t_a). If the CV values are predetermined and assumed to represent the whole population, the t statistic can be replaced by the normal distribution function standardized variate for the desired probability level, z_p. Setting CI=1, N=1 and the sign in the numerator negative (for the lower bound) and solving for $(\overline{Q} \cdot t_a)/(d_u + S)$ gives

$$\frac{\overline{Q} \cdot t_a}{d_u + S} = \frac{1 - z_p^2 \, CV_D^2}{1 - z_p^2 \, r \, CV_D CV_Q - z_p \sqrt{CV_Q^2 + CV_D^2 - 2rCV_Q CV_D - z_p^2 CV_D^2 CV_Q^2 (1 - r^2)}} \qquad (8)$$

If there is no covariance between the inflow and infiltration ($r = 0$) and the higher-order term is dropped, equation 8 simplifies to

$$\frac{\overline{Q} \cdot t_a}{d_u + S} = \frac{1 - z_p^2 \cdot CV_D^2}{1 - z_p \sqrt{CV_Q^2 + CV_D^2}} \qquad (9)$$

The CV of amount absorbed plus surface storage, CV_D, is related to the CV of cumulative infiltration, CV_I. As discussed before, since the criterion is to complete advance by a predetermined time, average IOTs will be equal on any furrows which just meet the criterion. With equal average IOTs, d is proportional to I and, thus, $CV_d = CV_I$. By the definition of the variance of a sum (and recalling that variance = $s^2 = (CV \cdot \overline{x})^2$),

$$CV_D \cdot (d_u + S) = [CV_d^2 \cdot d_u^2 + CV_s^2 \cdot S^2 \\ + 2r_s \cdot CV_d \cdot d_u \cdot CV_s \cdot S]^{0.5} \qquad (10)$$

where CV_s is the CV of surface storage and r_s is the correlation coefficient between S and d. Since the variance of surface storage will generally be small relative to the variance of the depth applied, the first term of equation 10 will dominate and CV_D can be approximated by

$$CV_D \approx \frac{d_u}{d_u + S} \cdot CV_d = \frac{d_u}{d_u + S} \cdot CV_I \qquad (11)$$

Note that when $CV_Q = 0$ and $\overline{Q} = Q_p$ (uniform inflow), and since $Q_u \cdot t_a = d_u + S$ (eq. 5), equation 8 reduces to equation 6. Also note that with $r = 1$ (inflow and infiltration perfectly correlated and $CV_Q = CV_I$) in equation 8, $(\overline{Q} \cdot t_a)/(d_u + S) = 1$ and no excess is required.

Figures 2 and 3 show the percent excess gross application ($100 \cdot ((\overline{Q} \cdot t_a)/(d_u + S) - 1)$) required to achieve completed advance on given portions of sets of furrows, P, with varying values of CV_D, CV_Q, r, and P. The required excess gross application is large if either CV_D or CV_Q is large. The excess is more sensitive to CV_Q because as CV_Q increases, the required $\overline{Q}$ increases so the absolute flow

variability ($s = CV_Q \cdot \overline{Q}$) increases more than CV_Q. Trout and Mackey (1988) found that inflow variability does in fact increase with the average flow rate.

Figure 2 also shows how correlation between inflow and infiltration decreases the required excess gross application. Correlation would result from the irrigator setting or adjusting the inflows for observed or perceived differences in advance rates or runoff (and thus infiltration) or by the flow rate in the furrow affecting infiltration rates. Trout and Mackey (1988) measured correlations generally below 0.3 when the irrigator had not consciously adjusted inflows and between 0.5 and 0.7 when he had. Adjusting the furrow inflows for observed advance or runoff differences to achieve a correlation of 0.7 reduces the required excess gross application by about 50%.

Figure 3 shows the effect of the required portion of furrows with completed advance on the required excess gross application. Due to the nature of the normally distributed values, meeting advance criteria on a large portion of the furrows is costly in terms of excess application and, thus, potential water loss. An irrigator's criterion for percent advance completion will normally increase through an irrigation. While a 90% advance completion halfway through an irrigation might be adequate, a 98% completion might be desired by the end of the irrigation. Although high advance criteria later in an irrigation increases the required excess gross application attributable to spatial infiltration and inflow variability, the runoff loss attributable to the decrease in infiltration rate with time after the advance criteria is met will simultaneously decrease. The farmer's advance criteria increases with time due to his experience with the effect of decreasing infiltration rates on advance. If the advance criterion is rationally chosen, the total runoff due to the combination of inflow and infiltration spatial variability and infiltration rate decreases should remain fairly constant through an irrigation.

The farmer's criteria for when advance should be complete (fraction of the total irrigation time) should depend upon how infiltration varies with time. For soils
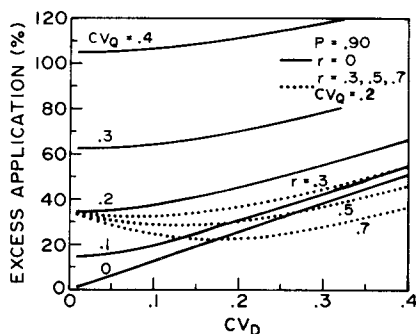

Figure 2–Excess gross water application required to complete advance on 90% of a set of furrows for varying levels of inflow and infiltration variabilities and correlation between inflow and infiltration.
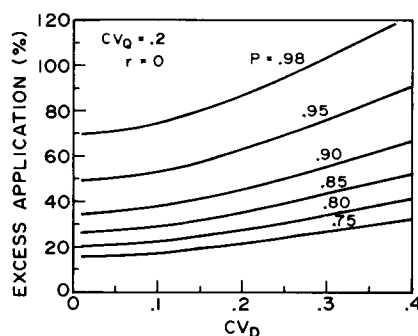

Figure 3–Excess gross water application required to complete advance on varying portions of furrow sets for varying infiltration variability ($CV_Q = 0.20$).
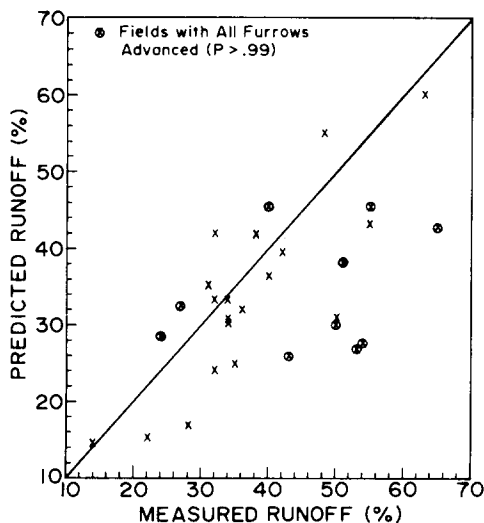
Figure 4–Runoff predicted by equation 8 vs. measured runoff from furrow-irrigated fields in southern Idaho.

with high initial and low final infiltration rates, completing advance by the end of the irrigation set is often adequate. However, if infiltration is dominated by the sustained or basic rate, net water application is nearly proportional to IOT, and the IOT at the tail end must be a large proportion of the irrigation time to achieve good application uniformity. These soils which require earlier advance completion and, thus, produce runoff for longer times generally produce lower average runoff rates because of their constant infiltration rates.

In the southern Idaho study area of Trout and Mackey (1988), infiltration into the silt loam soils quickly approaches a basic or steady-state value and surface storage in the relatively steep furrows is small. Thus, late in the irrigation event when measurements were made, $d_u \gg S$ so that $CV_D \approx CV_I$, the cumulative inflow and infiltration variability can be represented by the inflow and infiltration rate variabilities, and most runoff would be the result of the excess gross application required due to infiltration and inflow variability. Figure 4 shows predicted runoff (excess gross application calculated by eq. 8) vs. measured runoff for 30 fields based upon measured CV_Q, CV_I, and r values and the portion of the furrows with runoff, P. The diagonal line represents correct prediction of the measured runoff. Fields on which water was running off from all furrows (P = 1) could not be assigned a standardized variate value, so a probability value of 0.99 was used. These data are circled on the figure.

Although the scatter is large, which would be expected from the relatively small sample sizes ($\overline{N} = 40$) used to determine the distribution parameters, the trend is evident. Ignoring fields with all furrows having completed advance, the average runoff predicted by equation 8 is only four percentage points less than that measured. This small under prediction of runoff is expected since surface storage was ignored. Over half of the predicted values are within ±5 percentage points of that measured and 95% are within ±12 percentage points. Note that for most fields with all furrows advanced, runoff exceeded that predicted assuming P = 0.99, as would be expected.

EFFECT OF INFILTRATION VARIABILITY ON IRRIGATION TIME

Irrigation is normally continued until the irrigator perceives that adequate water has been absorbed on a desired portion of the field. This concept has been discussed by several authors (i.e., Till and Bos, 1985; Hart and Reynolds, 1965). With variable infiltration, each area of the field does not infiltrate water at the same rate and thus will not have absorbed the same depth of water at any given time. Thus, extra water must be applied by extending the irrigation time to ensure an adequate portion of the field, or in this case, furrows, has absorbed adequate water. Extending the irrigation time to compensate for water distribution differences down the furrow due to IOT differences is a common management practice. For example, irrigation is often continued until the tail ends of furrows have absorbed adequate water. Furrow-to-furrow infiltration variability would require additional irrigation time to insure that the tail ends are adequately irrigated on a desired portion of the furrows.

With sprinkler or drip irrigation, quantifying water distribution, which is assumed dependent only on the application system, is straightforward. However, with surface irrigation, water distribution is difficult to quantify because water absorption is dependent on the interaction between infiltration rate and IOT, and IOT is dependent on infiltration rate plus other factors. Clemmens (1988) describes distribution of these two parameters as both random variability and deterministic trends both across and down the field. He statistically combines the parameter variabilities to estimate water distribution uniformity. However, as Clemmens points out, quantifying the interactions (covariance) between variable infiltration and IOT is very difficult.

In the present work, by making several simplifying assumptions, the interactions between furrow-to-furrow infiltration variability and IOT are quantified allowing evaluation of the effect of infiltration variability on irrigation performance and management decisions. These assumptions include: 1) no infiltration variability down individual furrows; 2) only random infiltration variability
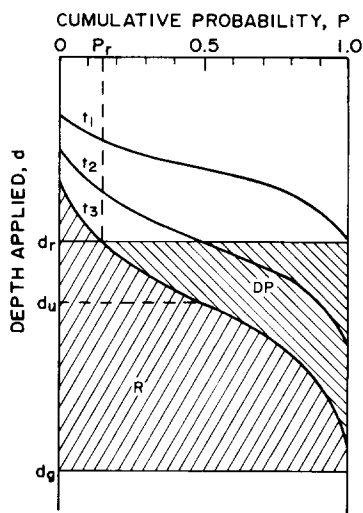

Figure 5–Cumulative distribution of water infiltrated at three application times. Areas DP and R represent deep percolation and runoff losses, respectively.

among furrows; 3) relative infiltration variability (CV_I) is constant with time; and 4) furrow inflows are uniform.

To simplify the analysis initially, assume no IOT differences as would be the case near the head-end of the furrows where the advance (and recession) is essentially instantaneous. Figure 5 shows a distribution of head-end water application at three progressively increasing times. Since IOT is assumed equal to the irrigation time, these net water application distributions are equal to the cumulative infiltration distributions ($CV_d = CV_I$). As time progresses, the amount of absorbed water increases and thus the curves move downward. The curves shown depict constant relative infiltration variability with time (CV_I = constant). If the variability changes with time, the shape of the curves will change. For example, if initial infiltration is more variable than the final rate, then the distribution will tend to flatten (become more uniform) with time.

In Figure 5, the required application depth is denoted d_r. Although the average furrow absorbs adequate water by time t_2, half the furrows are still under-irrigated at that time. If the irrigator wishes that no more than P_r portion of the furrows receive less than the required depth, the irrigation must continue until time t_3. By that time, the average amount of water absorbed would be d_u and the relative excess application required due to infiltration variability would be d_u/d_r. Assuming a normal distribution, the excess application for an allowable portion of the furrows left deficient, P, would be (from eq. 4)

$$\frac{d_u}{d_r} = \frac{1}{1 + CV_I \cdot z_p} \qquad (12)$$

Note that most texts only list the positive portion of the symmetrical normal cumulative distribution function (0.5<P<1.0). For P<0.5, $z_p = -z_{(1-P)}$ (for example, $z_{0.2} = -z_{0.8} = -0.84$).

The excess water application depicted is lost both to deep percolation on the l-P portion of the furrows with higher infiltration rates and to tailwater runoff. The deep percolation and runoff loss are denoted graphically in figure 5 by areas DP and R, respectively, for a gross application (total inflow) d_g.

Of course, irrigation adequacy decisions are not normally based on conditions at the head end of the field but on conditions near the tail where IOT is less. There the analysis is more difficult because infiltration variability affects both the infiltration rate and IOT. Furrows with lower than average infiltration rates have faster than average advance rates and thus relatively longer IOT at the tail. Conversely, furrows with higher than average infiltration rates have slower than average advance rates, but once they complete advance, the application depth tends to catch up with the other furrows due to their higher infiltration rates. This interaction between infiltration and advance rates reduces the variability of water application at the tail end compared to that at the head.

To estimate the effect of the interaction between infiltration rate and IOT on water application distributions down furrows, water distributions were projected for sets of furrows with varying infiltration rates with a kinematic wave surface hydraulics model*. Infiltration was modeled by the extended Kostiakov relationship

$$I = Kt_o{}^a + Ct_o \qquad (13)$$

where I is cumulative infiltration, t_o is infiltration opportunity time, and K, a, and C are empirical coefficients. Coefficients were chosen for the infiltration function for the average (P = 0.5) furrow (furrow with the average infiltration characteristic). Then K and C values were calculated which would produce several selected cumulative infiltration probabilities assuming a normal distribution of furrow infiltration with $CV_I = 0.25$. For example, if the K value for P = 0.5 (average furrow) is $K_{0.5}$, the K_p value for the probabilities, P, were calculated as

$$\frac{K_p}{K_{0.5}} = 1 + CV_I \cdot z_p \qquad (4a)$$

Infiltration variability was assumed constant with time (i.e., coefficients K and C were perfectly correlated ($C_p/C_{0.5} = K_p/K_{0.5}$) and a was constant). By inputting the generated series of infiltration relationships into the model, water application distributions down furrows were calculated for a series of furrows representing a range of infiltration probabilities. All other model inputs were held constant (furrow slope = 0.005, Manning's roughness = 0.02, furrow length = 381 m, furrow spacing = 1 m, inflow = 1 L/s).

Two average infiltration relationships which bracket a wide range of conditions were used. The first represents a soil with a high initial but continually decreasing infiltration rate (K = 28 mm/h^a, a = 0.4, C = 0). The second is dominated by the basic or steady-state infiltration rate (K = 7 mm/h^a, a = 0.3, C = 5 mm/h). Both functions result in 75-mm cumulative infiltration after 12 h. The first infiltrates 50% of this total in the initial 2 h while the second infiltrates only 25% in that elapsed time. The furrow with these average infiltration characteristics will be termed the median furrow.

Water absorbed by four specific portions of each furrow vs. the cumulative probability of occurrence of a furrow
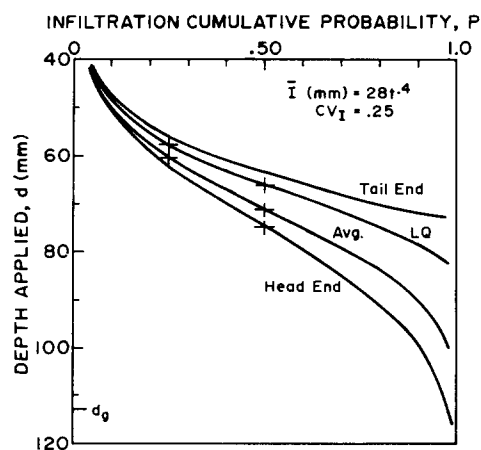

Figure 6–Distribution of water absorbed by the head end, tail end, low quarter (LQ), and whole furrow (Avg) for a rapidly decreasing infiltration relationship with the furrow-to-furrow $CV_I = 0.25$.
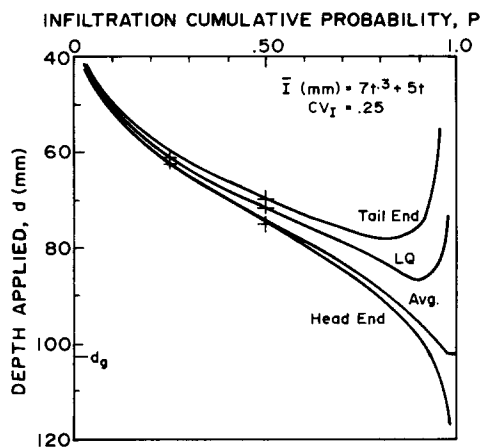
Figure 7–Distribution of water absorbed by the head end, tail end, low quarter (LQ), and whole furrow (avg) for a relatively constant infiltration relationship with the furrow-to-furrow $CV_I = 0.25$.

with that infiltration characteristic is plotted in figures 6 and 7. Shown are the distributions of water applied during a 12-hour irrigation 1) at the head (inflow) end of the field, 2) at the tail (outflow) end, 3) by the low quarter (LQ), and 4) the average net application (avg) for the whole furrow. The low-quarter application is the average application to that 25% of the furrow length with the least application (in this case, the tail 25%). The head-end applications follow the normal infiltration distributions. As expected, the furrow-to-furrow application distributions flatten (become more uniform) at locations farther from the head-end of the furrow. The amount of flattening increases with IOT differences. Note that the plotted curves are not cumulative distributions in all cases (see the LQ and tail-end curves in fig. 7), but are application depths plotted vs. the infiltration cumulative probability.

With the decreasing infiltration relationship (fig. 6), advance on the median furrow required 4.5 h, resulting in the 75-mm head-end application being 14% greater and the 71-mm average application being 7% greater than the 66-mm low-quarter application (DU-71/66 = 0.93). However, on 25% of the furrows (P = 0.25), the low-quarter application was less than 58 mm. If the target were to meet an application requirement on 75% of the low quarters, then 14% extra water had to be applied due to the among-furrow infiltration variability (i.e., $d_u/d_r = 66/58 = 1.14$ for P = 0.25 for the low quarter). Note that this is less than the 20% excess which would have been predicted by equation 12 at the head end (and by the infiltration rate variability) because the low quarter application is more uniform.

The application distributions for the more constant infiltration relationship are shown in figure 7. Because of lower initial infiltration rates and in spite of a 10% smaller inflow rate (inflow = 0.9 L/s), the advance is more rapid on the median furrow (70 min) than in the previous example. Thus, both the head-end and average applications to the median furrow are only about 4% greater than the 72-mm low-quarter application (DU = 0.96). In spite of the rapid advance on the median furrow, advance was not complete after 12 h on 2% of the furrows. The slow advance on the furrows with the highest infiltration rates (right side of figure) is reflected in the rapid decrease in the low-quarter and tail-end applications. Excluding those slow-advance

furrows, 25% of the furrows received less than 61 mm in the low quarter so the excess application due to variability necessary to exceed the low-quarter requirement on 75% of the furrows was 18%, quite close to the 20% predicted by equation 12 based on CV_I.

The total excess net application required to exceed low-quarter criteria on 75% of the furrows ($d_{avg}(P = 0.5)/d_{LQ}(P = 0.25)$) was about 23% for both infiltration relationships. The nature of the infiltration relationship primarily determined the portion of the excess attributable to IOT differences down the furrows (i.e., d_{avg}/d_{LQ} of the median furrow) and the portion caused by furrow-to-furrow infiltration variability (i.e., d_u/d_r of the low-quarter distribution). With the decreasing infiltration relationship (fig. 6), 62% of the excess is due to furrow-to-furrow variability while in figure 7, this among-furrow variability is responsible for 79% of the required excess application.

These two cases demonstrate the important influence of furrow-to-furrow infiltration variability on water distribution uniformity. Approximately 10% of the furrow length receives less than the low-quarter application to that furrow. However, for a furrow-to-furrow infiltration CV of 25%, much more than 10% of the field area receives less than the low-quarter application for the furrow with an average infiltration characteristic. For the two examples given here, 30% and 42% of the head ends received less than the low-quarter application to the median furrow. If the adequacy criterion is to exceed the requirement on 90% of the furrows rather than on 90% of the median furrow, the required excess application would have been 40% vs. 8% in figure 6 and 42% vs. 4% in figure 7. To base irrigation distribution estimates only on IOT differences is ignoring the more critical problem of infiltration variability.

DIFFERENTIATING LOSSES

As depicted in figure 5, the area below the required application, d_r, but above the distribution of average net applications, represents the deep percolation loss. The area between the average applications and the gross application, d_g, represents the tailwater runoff loss. If the water application distribution can be defined mathematically, these losses can be quantified.

For normal application distributions, runoff can be calculated as (adapted from Warrick et al., 1989, Table 1, column 3)

$$R = d_u \cdot CV \, [(2\pi)^{-.5} \cdot \exp(-z_g^2/2) + z_g \cdot P(z_g)] \quad (14)$$

where R is the runoff expressed as an equivalent depth, and

$$z_g = (d_g - d_u)/(d_u \cdot CV) \quad (14a)$$

Likewise, deep percolation loss can be calculated as

$$DP = d_u \cdot CV \, [(2\pi)^{-.5} \, [\exp(-z_r^2/2) - \exp(-z_g^2/2)] + z_g \cdot (1 - P(z_g)) - z_r(1 - P(z_r))] \quad (15)$$

where DP is deep percolation expressed as an equivalent depth, and

$$z_r = (d_r - d_u)/(d_u \cdot CV) \quad (15a)$$

Note that z_g and z_r could also be determined from desired probability levels $P(z_g)$ and $P(z_r)$.

EXAMPLE

The distribution of average application shown in figure 6 is close to a normal distribution with CV = 0.22. For the 12-h application shown, $d_g = 112$ mm and $d_u = 71$ mm. Assuming $d_r = 60$ mm, then $z_r = -0.70$ and $z_g = 2.62$. Normal distribution function tables give $P(z_r) = P(-0.70) = 1-P(0.70) = 0.26$ and $P(z_g) = P(2.62) = 0.996$. Substituting these values into equations 14 and 15 yields R = 41 mm and DP = 13 mm. Total storage (assuming d_r = the soil profile storage capacity) would be d_g - R - DP = 58 mm or 2 mm less than the requirement, due to the deficiency remaining on the furrows with low infiltration.

CONCLUSIONS

The consequences of furrow-to-furrow inflow and infiltration variabilities are tailwater runoff and deep percolation losses while a portion of the field receives inadequate water. Furrow-to-furrow infiltration variability in combination with inflow variability causes an irrigator to increase inflow rates to achieve a desired advance time on a desired portion of the furrows. Infiltration variability also causes an irrigator to irrigate longer to achieve adequate net application depths on furrows with low infiltration rates. Furrow-to-furrow infiltration variability will generally cause more water application variability than IOT differences along furrows. A furrow irrigator generally must over-irrigate by at least 30% if he wishes to apply adequate water to over 80% of the field due to these variabilities.

Even with irrigation scheduling or soil moisture monitoring to indicate the correct average requirement and cutback inflows to match decreasing infiltration rates, the furrow irrigator still must over-irrigate to attain high crop yields. He is faced with the practical management decision of choosing between an acceptable amount of water loss (and the nitrogen loss which accompanies deep percolation) and the portion of the field he is willing to leave under-irrigated. The consequences can be statistically quantified if the infiltration variability is known. Only by reducing infiltration and inflow variabilities and by collecting and reusing tailwater runoff can he irrigate efficiently without sacrificing yield.

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